Stability Analysis of the Walking Beam Vehicle

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***Abstract*—The walking beam vehicle is one of several concepts being considered by NASA for an unmanned Mars rover vehicle. It is a simple walking machine with either eight or nine controlled degrees of freedom depending on the configuration used. Martin Marietta Corporation proposed the concept. This paper contains an analysis of stability, and available leg stroke on sloping terrain.**

1. INTRODUCTION

The walking beam is one of several concepts being considered for an unmanned Mars rover [1 to 3]. It is a simplified walking machine (Figure 1).

The vehicle has two main members called the "beam" and the "body." These two members can slide relative to one another, by means of a roller and track assembly. They can also rotate relative to one another, by means of a turntable.

The beam carries three legs arranged at the vertices of an equilateral triangle with sides of length 6 m. The body carries either three legs, arranged at the vertices of a triangle of side 3 m, or four legs arranged at the vertices of a square of side 2.45 m. All legs telescope in a direction normal to the plane of the member on which they are mounted. They are otherwise rigidly mounted to the two main members of the machine.

Several different configurations of the walking beam have been studied. Configurations considered also include versions with the beam legs having twice the extension of the body legs, as well as versions with all legs having the same extension. The intent here is to maintain the body in a level attitude on most slopes. A modification in the design of the body has been suggested [4]. The suggested configuration of the body has the shape of a square, with four legs at its corners. The diagonal of the square is equal to the diameter of the circumcircle of the triangular body. This is necessary to avoid increased interference with the beam legs.

There is an infinite number of possible ways of controlling the attitude of the vehicle when it is moving on a steep slope. Two particular strategies represent the extremes of a spectrum. If the body remains parallel to the slope, the full stroke of the sliding joint is kinematically available at all times. However, the requirement of static stability diminishes both the slopes which can be traversed and the available stroke. Alternatively, the body may remain in a horizontal attitude at all times. This removes the stability problem but the available stroke for locomotion either up or down the slope is limited by interference with the terrain. This is due to the limited available extension of the legs. The purpose of the study reported here was to explore the relationship of available stroke and stability margin to body attitude. This relationship is studied over the range of slopes on which the vehicle can theoretically operate. The configuration studied is one in which the extension of the beam legs is twice that of the body legs. This configuration helps to keep body level over a wide range of terrain gradients. Both three and four-legged body configurations are studied. An earlier study by Price [1] analyzed the stability of three and four legged body configurations with all legs of equal length. It did not include analysis of the available stroke.

Figure 1. The Martin Marietta Walking Beam Mars rover vehicle. The major dimensions of the vehicle after deployment are shown. The T shaped member is called the "beam." The smaller triangular member is the "body." The body translates relative to the beam along a track on the spine of the T. There is also a turntable with axis at the body centroid permitting the body to rotate relative to the beam.

Walking on a sloping terrain increases the problem of stability of the vehicle. Since the vehicle travels at a very low speed, only static stability is important. The stability

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margin, which gives a measure of the stability of the walking beam, is a function of the leg set supporting the vehicle, the attitude of the vehicle, the height of the center of gravity, the relative positions of the centers of the beam and the body, the direction of motion on the slope, and the gradient of the slope.

The parameter of specific interest, during the motion of the vehicle, is the stroke available for the next step. The physical limitations of the vehicle, and the requirement for stability during the complete step cycle, limits the stroke.

In this paper an analysis is carried out individually for the beam and for the body, and then for the whole vehicle. The factors affecting the stability of the rover are discussed and the characteristics of three and four legged bodies are compared.

2. METHOD OF DETERMINING STABILITY

Several different methods are available to determine the static stability of a vehicle. In the following study, in order to to evaluate the degree of stability, the method of projecting the center of gravity into a plane passing through the points of foot contract is used.

The support polygon is the figure formed by joining the points of contact. The vehicle is stable as long as the projection of the center of mass lies within the support polygon. The minimum distance between the projected joint and the boundary of the polygon is the stability margin. The stability margin gives a measure of the degree of stability. If the stability margin drops below zero, the vehicle becomes unstable.

As shown in Figure 2, the effect of a slope on the stability margin is an effective shift, in the downslope direction, of the projection of the center of mass in the plane of the feet. This is indicated by x_1. The limiting condition for stability is when the stroke, x_2, which would carry the center of mass to the boundary of the support polygon, if the vehicle were on level ground is equalled, or exceeded, by x_1. This is the condition used to derive the limiting stability conditions 3 and 4 used below.

In some circumstances, the available stroke is not limited by stability, but is limited by interference either between the body and beam structures or legs, or between the legs, at maximum retraction, and the ground. This case can be handled in the same manner, with x_2 interpreted as the available stroke before interference occurs. The limiting conditions 1 and 5 are of this type.

3. TRIPOD STABILITY ANALYSIS

Figure 2 shows the tripod with an orientation angle α. The analysis is carried out by varying the height of the center of mass as a function of the height of the legs. The angle of slope θ for which the tripod will roll over is determined for various orientations. This happens when distance x_1 exceeds x_2. The results for the smaller tripod: the body, and the outer tripod: the beam, are given in Figure 3.

The beam tripod becomes unstable only for very high gradients. For slopes of less than 30°-35°, the beam provides a very stable base. The body stance is the potential candidate for the problem of stability. The dimensions of the body are such that it does not give enough stability in some orientations even for slopes of 30°. The situation can deteriorate when the whole vehicle is considered, with the relative position of the beam shifting the combined center of mass away from the centroid of the body triangle. This combination is analyzed in the present section.

4. FACTORS AFFECTING THE STABILITY OF THE VEHICLE

Various factors affect the stability of the vehicle which are required to define the complete configuration of the vehicle and the terrain. These factors are discussed in this section, together with their effects on the vehicle stability.

A. *Legset*

Each leg set when supporting the rover gives a support pattern in the form of a triangle, or a square. When supported by the beam leg set, the supporting triangle has a minimum side of 6 m which provides a highly stable base. In comparison, the three-legged body leg set gives a smaller support pattern, making the vehicle more prone to instability. The center of mass is designed to lie within the body triangle for the complete stroke. Thus, on level terrain, support on the body leg set is stable for the whole vehicle.

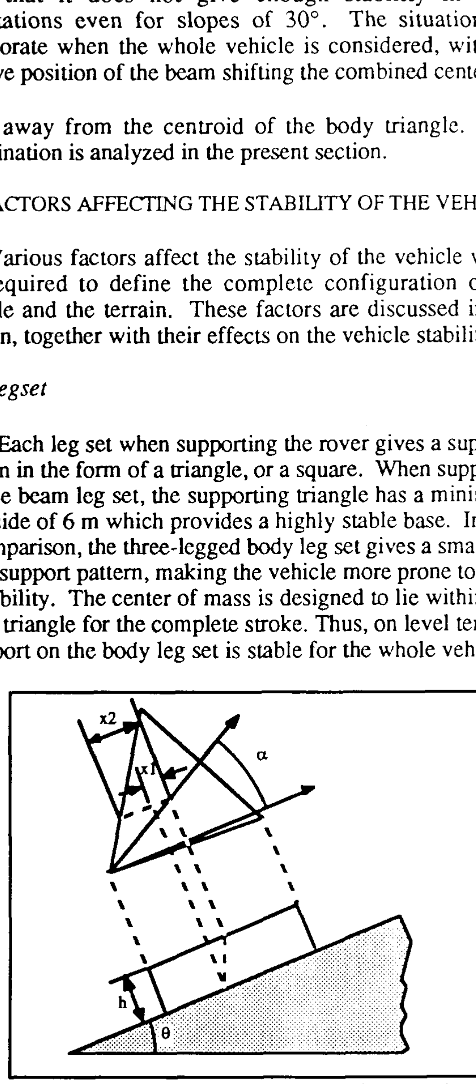

Figure 2. Stability analysis of a general equilateral tripod with plane parallel to the slope. Stability is a function of the slope angle, θ, the height of the center of mass, h, and the orientation of the vehicle relative to the direction of steepest slope, α. The upper part of the figure is the projection of the tripod in the plane of the slope.

On sloping terrain the vehicle may become unstable for extreme stroke positions. Stability as a function of terrain gradient, and direction relative to the maximum slope line may be plotted as shown in Figure 3. The corresponding diagram for beam support is generally similar to Figure 3a, but the gradient angles are approximately 50% greater.

Figure 3b gives the gradient for which the body would become unstable with the four-legged body config-uration. It can be seen from the diagram presented that a square shaped body will be more stable.

B. Height of The Center of Mass

The height of the center of mass is a function of the heights of the legs supporting the rover. Figure 4 gives the effect of the height of the center of mass on the stability of the rover. The stability margin decreases as the leg heights increase, so that in any configuration the height of the center of mass should be minimized. The lower center of mass also decreases the rate at which instability will set in, giving time to recover, if the stability margin goes to zero at any time.

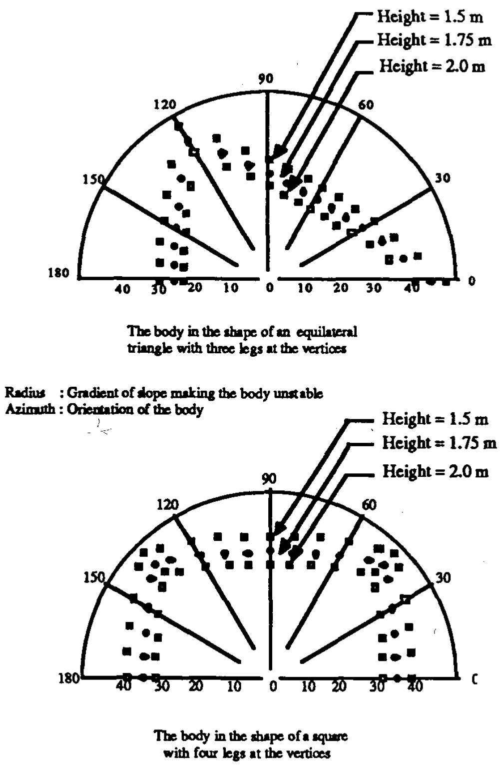

Figure 3. Stability as a function of direction of motion for different heights of the center of mass for (a) three-legged body configuration and (b) four-legged body configuration.

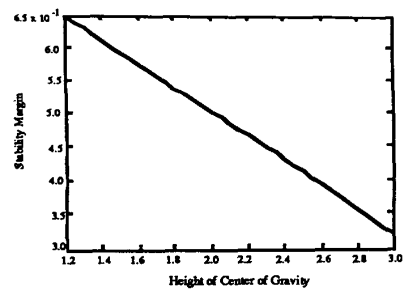

Figure 4. Effect of the height of the center of mass on the stability of the rover

C. Attitude of Vehicle

Depending on the heights of the legs supporting the rover, the attitude of the vehicle can be changed. Figure 5 shows two extreme ways of climbing a slope. In case 1, the attitude of the vehicle is parallel to the slope, making the complete stroke available at all times, but the configuration can become unstable for steep slopes. In case 2 the attitude is horizontal at all times. This eliminates the problem of instability but may result in restricted available stroke on steep slopes. This reduces the distance which the vehicle may cover in each step cycle.

Both methods are adequate for walking on gradual slopes but steeper slopes, require a compromise. In a high gradient situation the vehicle should maintain an attitude which allows a large stroke and maintains stability of the rover at all times. The leg lengths determine the range of attitudes which are attainable. With the beam supporting the rover the attitude can vary from -28° to +28°, measured from the angle of slope. The range of possible attitude angles allowed by the body legs is from -24° to +24°, measured from the angle of slope.

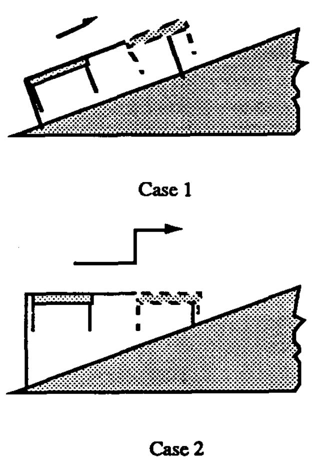

Figure 5. Two extreme strategies for climbing or traversing slopes. In the first case the body of the machine is parallel to the slope. In the second the body is horizontal at all times.

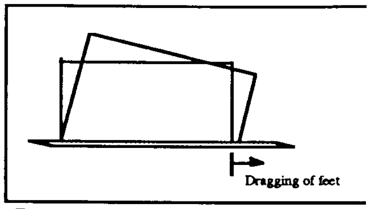

Figure 6. Dragging of feet with change of attitude. Some scuffing is necessary with this geometry whenever the attitude changes relative to the ground slope. It is desirable to minimize scuffing to lessen the likelihood of significant slip and to reduce energy consumption.

The legs are perpendicular to the plane of the vehicle and are mounted on the body and the beam by solid joints. As shown in Figure 6, change of attitude requires some dragging of the foot on the ground. Since this may result in gross slipping, frequent changes in attitude are not desirable. Therefore, while traversing steep slopes the same body attitude should be maintained until there is a large change in the slope gradient.

The graph in Figure 7 illustrates the effect of the attitude of the vehicle on the stability margin.

D. Orientation of the Support Pattern

Since the support pattern is a triangle, the stability margin is a function of the orientation of the support pattern. Figure 8 shows the effect of a change of orientation on the stability margin. For the case of beam leg support, the direction of motion restricts the orientation of the supporting triangle. However, in the case of body support, the support pattern can be rotated so as to have a suitable orientation to stabilize the vehicle.

E. Relative Motion of The Center of Mass

The relative motion of the combined center of mass due to the motion of either the beam or the body when the other is supporting the rover is of critical concern. This may limit the stroke during each motion cycle. As was mentioned above, body support gives a smaller support pattern. Therefore, it is during this phase of motion that the problem of tip over may become serious and only a small stroke may be allowed. In order to decrease the motion of the center of mass during this phase, the mass ratio of the body to the beam is made to be greater than 1. In the next section an analysis of this problem is presented in more detail.

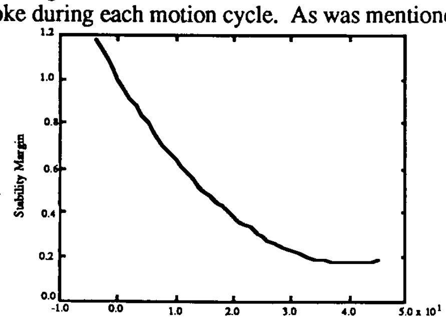

Figure 7. Effect of body attitude on stability margin. The attitude angle is the angle between the plane of the body and the ground slope.

A second strategy to address this problem is the use of a body with four legs, thereby increasing the size of the support pattern.

5. ALLOWABLE STROKE

From the previous section, walking on a slope requires the maintenance of a proper attitude of the rover to allow large strokes without instability occurring at any time during the motion cycle. The attitude is the angle made by the plane of the rover with the slope. Further, the leg lengths are controlled so that the normal to the plane of the rover lies in the plane normal to the slope. In this section we discuss the stroke which is allowed in different directions of motion, for both phases of the motion cycle, as a function of attitude.

The requirement of stability limits the stroke, requiring the complete motion of the projection of the combined center of mass to be within the support pattern. The other factors restricting the stroke are the physical and geometrical limitations of the track length and the length of the legs. The configurations just before and after changing the leg set are important for deciding the available stroke.

For the purposes of this analysis the following variables are introduced: h is the height of center of mass; α is the attitude angle, measured from the horizontal plane; θ:is the angle of slope of the terrain, also measured from the horizontal plane; β is the direction of motion, measured from the direction of steepest slope; R_0 is the radius of the circumcircle of the body triangle (R_0 = 1.733m); R_c is the radius of the circumcircle of the beam triangle (R_c = 3.467m); D is the distance between the centroid of the beam and that of the body

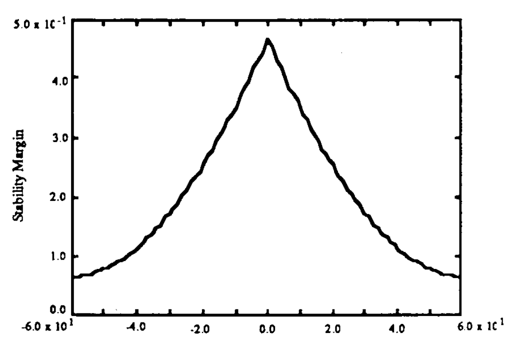

Figure 8. Stability margin as a function of orientation relative to the maximum slope direction.

The height of the center of mass is minimized in all configurations. The center of mass is assumed to lie in the central plane of the body.

A. *Movement Upslope*

In case of the rover moving upslope the instability can occur if the projection of the combined center of mass undershoots the lower boundary of the support polygon. During the relative movement, the forward motion of the center of mass stabilizes the rover as its projection moves into the support polygon away from the lower boundary. Therefore, the critical time is when the moving part is at the extreme rear position, i.e., at the beginning of the stroke. In order to ensure that the rover is stable at all times the stroke should be limited so that the next support configuration is initially stable, when the leg set supporting the rover changes.

(i) Beam Movement

After beam movement, the beam leg set supports the rover in the next phase of the motion cycle. The beam provides a large support polygon which gives a highly stable base and the problem of instability only occurs for very large angles of slope. The overriding concern is the limitation of the stroke due to the attitude of the rover which may allow only restricted motion in the forward direction before the beam feet strike the ground

The stability of the next phase is governed by the set of conditions 1:

$$x_2 \geq x_1 \quad \text{where} \quad x_1 = h \sin\alpha \cos(\theta - \alpha) / \cos\theta,$$

$$x_2 = (R_C - 2D/3) \sin 30° / \sin(150° - \beta) \qquad (1)$$

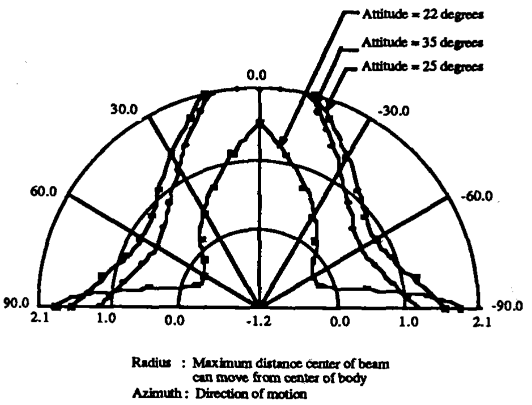

Figure 9. Maximum available stroke as a function of direction of motion for different body attitudes in body support.

Equation 2 gives the movement allowed before the farthest forward beam leg touches the ground:

$$D = (d - R_0 \cos\gamma - R_C \cos(60° - \beta)) / \cos\beta \qquad (2)$$

where d is $(h_2 - h_1) / \tan(\theta - \alpha)$, and h_2 is the maximum length of the body leg, and h_1 is its minimum. γ is the orientation angle of the body.

The minimum solution for D in Equations 1 and 2 is the maximum distance the center of the beam can travel from the center of the body. Figure 9 gives results, at various attitudes of the rover, for a slope of gradient 35°. Equation 1, alone, governs the solution for D until the difference between the attitude and the gradient becomes more than 12°, after which the allowable stroke decreases rapidly.

(ii) Body Movement

The leg lengths of the beam are designed so that the body can utilize the complete designed stroke at any allowable attitude of the rover when it is supported by the beam leg set. Therefore, the governing concern is that of the stability at the beginning of the next phase when the body will support the vehicle.

The maximum distance the body center can travel from the center of the beam is governed by the set of conditions 3:

$$x_2 \geq x_1 \quad \text{where} \quad x_1 = h \sin\alpha \cos(\theta - \alpha) / \cos\theta,$$

$$x_2 = R_0 \cos\phi - D \cos\beta / 3. \quad \phi = \sin^{-1}(d / 3R_0) \sin\beta. \qquad (3)$$

The maximum distance the body center can travel from the center of the beam, for various attitudes of the vehicle, while climbing a slope of given gradient in different directions can be plotted from these conditions in the same manner as Figure 9 [5].

B. *Movement Downslope*

The problem of instability during downward motion can occur if the projection of the combined center of gravity overshoots the boundary of the support polygon on the lower side. In contrast to the movement upslope, the forward motion of the center of mass moves its projection towards the lower boundary. Therefore, the critical time is when the moving part is at the extreme forward position. In order to ensure the stability of the vehicle at all times, the stroke is limited. The complete motion of the projection of the combined the center of mass is restricted to be within the support polygon. In order to achieve stable motion with maximum stroke, the attitude is considered to vary from the negative of the slope angle to zero degrees.

(i) Beam Movement

Stability considerations limit the stroke for beam movement downslope since the body leg set supports the rover giving a smaller support pattern. The stability margin depends on the orientation of the body which is independent of the direction of the motion. The orientation of the body may prohibit motion in some directions altogether for large slope angles.

The maximum distance center of the beam can travel from the center of the body is governed by the set of conditions 4:

$x_2 \geq x_1$ where $x_1 = h \sin\alpha \cos(\theta - \alpha) / \cos\theta$,

$$x_2 = (R_0 \cos(\beta - \gamma) - D/3) \cos(\gamma + \beta) / \cos\alpha \quad (4)$$

and γ is the orientation angle of the body.

The results given by these conditions may also be plotted in the format of Figure 9 [5].

(ii) Body Movement

In this phase of the motion the beam leg set supports the rover giving a stable base. The possibility of the rover becoming unstable during forward motion of the body arises only for large gradients. The lengths of the body legs impose the other limit on the motion of body. This happens only when large differences between the attitude and the slope gradient are present. At the end of the stroke the body legs should be able to reach the ground to support the rover for the next phase of the motion cycle.

The maximum distance the center of the body can travel is governed by the Equations 5 and 6

$x_2 \geq x_1$ where $x_1 = h \sin\alpha \cos(\theta - \alpha) / \cos\theta$,

$$x_2 = (R_C/2 - 2D/3) / \cos\beta. \quad (5)$$

$$D = 3(d - R_C\cos\alpha - R_0/2) / (2\cos\alpha) \quad (6)$$

where $d = (h_2 - h_1) / \tan(\theta - \alpha)$, and h_2 is the maximum length of the body leg, and h_1 is its minimum.

The minimum of the solution for D, in the Equations 5 and 6 gives the maximum distance that the center of the body can travel from the center of the beam. A plot is given in reference [5].

There is no apparent effect on the maximum stroke permissible during motion planning while traversing gradients since the radius of the body circumcircle remains the same. During the search for stable footholds on high gradients, more orientations of the body will be allowed a can be tested. This may permit somewhat greater stroke.

6. SUMMARY

The problem of ensuring the stability of the walki beam while it is moving on a gradient is discussed in tl paper. The walking beam can be supported in two wa during a motion cycle, either by the beam leg set or the bo leg set. Each of these stances gives a large enough base that the complete motion of the projection of the combin center of mass remains within the support polygon f motion on level terrain. However, on slopes, the rover m be unstable at the extremes of the stroke.

The attitude of the rover is critical while moving or slope. As the attitude angle approaches zero, correspondi to a level body, the stability of the rover increases, but t available stroke may diminish drastically. The attitude ca not be changed frequently since a change requires sor dragging of the placed feet that may trigger gross slippir Therefore, an optimal attitude which gives sufficient stabil and maximum stroke is selected whenever there is a lar change of gradient. A simple set of rules for controlling t vehicle attitude has been reported elsewhere [5].

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